石家莊鐵道大学
SHIJIAZHUANG TIEDAO UNIVERSITY

在线开放课程

理论力学

力对点之矩与力偶

力对点之矩与力对轴之矩

主讲 ： 郭树起

力对点之矩

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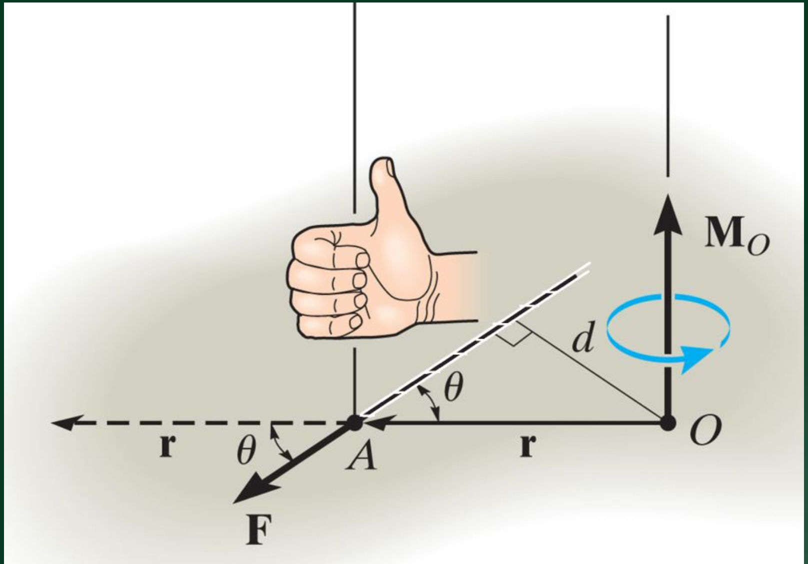
力对点之矩

力对点之矩的矢量表示

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$

大小：F(rsinθ)=Fd

方向：右手法则

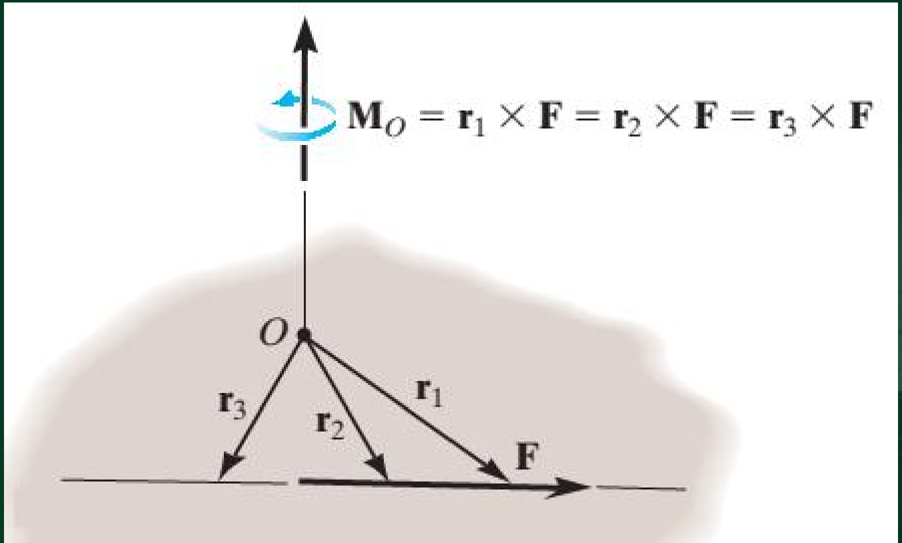


力对点之矩

力对点之矩的矢量表示

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$

$r_1 \sin\theta_1 = r_2 \sin\theta_2$
$= r_3 \sin\theta_3 = d$

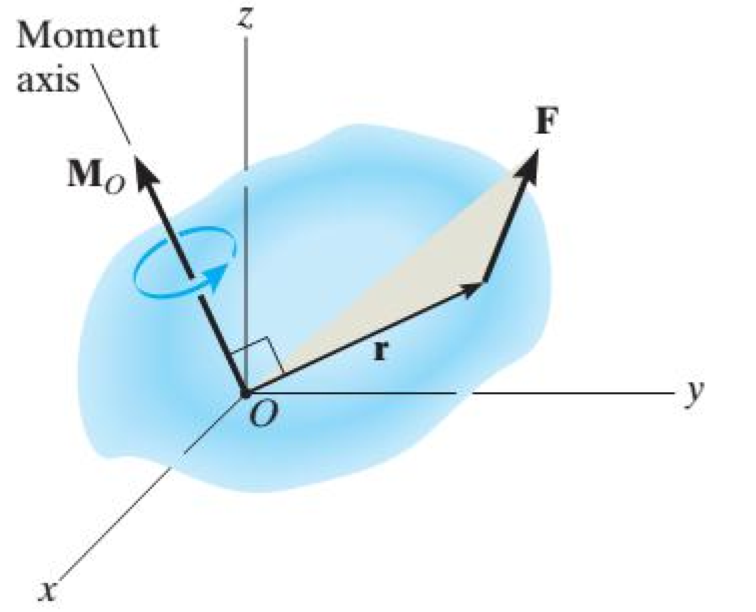


力对点之矩

力对点之矩的矢量表示

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

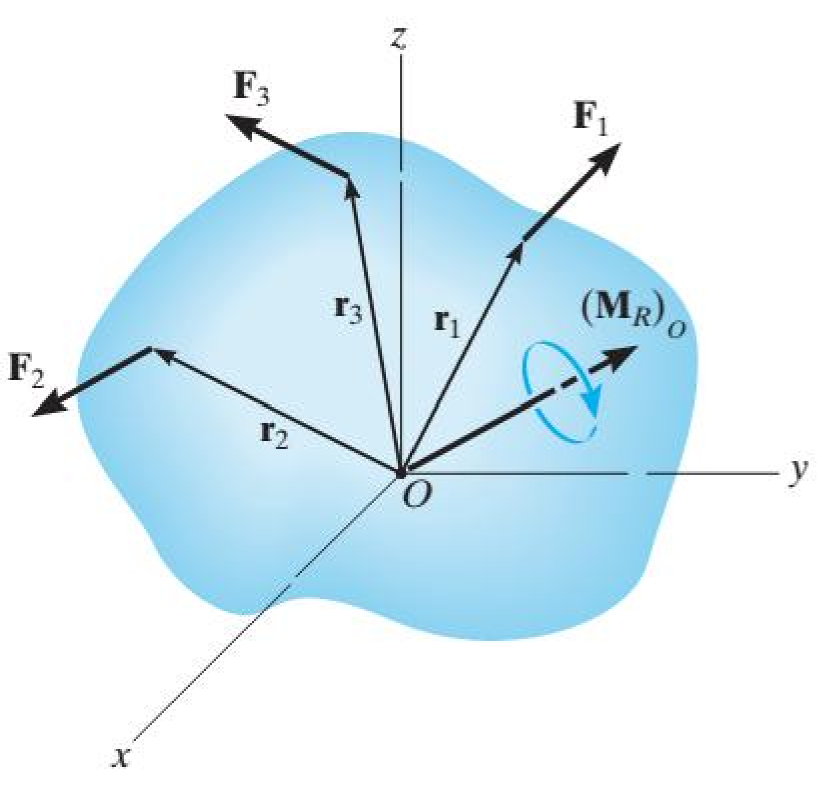


$$\mathbf{M}_O = (r_yF_z - r_zF_y)\mathbf{i} - (r_xF_z - r_zF_x)\mathbf{j} + (r_xF_y - r_yF_x)\mathbf{k}$$

力对点之矩

力系对点之矩

$$\vec{M}_O = \sum \vec{r}_i \times \vec{F}_i$$



力对点之矩

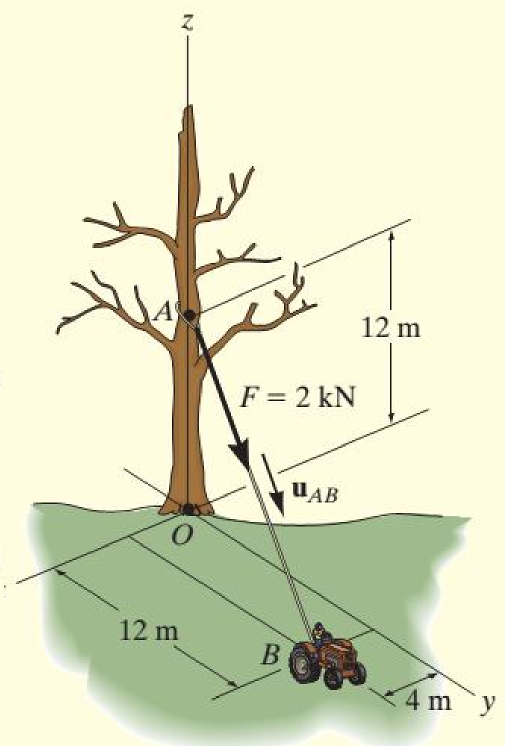
例：求力F对O的矩。u_{AB}是单位向量。

解：$\vec{M}_O = \vec{r}_A \times \vec{F} = \vec{r}_B \times \vec{F}$

$\mathbf{r}_A = \{12\mathbf{k}\}\ \text{m} \quad \text{and} \quad \mathbf{r}_B = \{4\mathbf{i} + 12\mathbf{j}\}\ \text{m}$

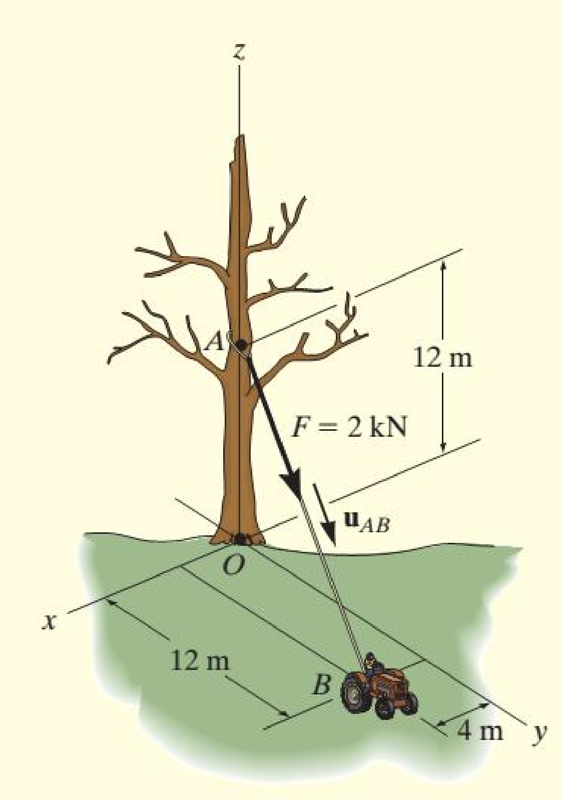
$$\mathbf{F} = F\mathbf{u}_{AB} = 2\ \text{kN}\left[\frac{\{4\mathbf{i} + 12\mathbf{j} - 12\mathbf{k}\}\ \text{m}}{\sqrt{(4\ \text{m})^2 + (12\ \text{m})^2 + (-12\ \text{m})^2}}\right]$$

$$= \{0.4588\mathbf{i} + 1.376\mathbf{j} - 1.376\mathbf{k}\}\ \text{kN}$$



力对点之矩

例：求力F对O的矩。u_{AB}是单位向量。



解：

$$\mathbf{r}_A = \{12\mathbf{k}\}\ \text{m} \quad \text{and} \quad \mathbf{r}_B = \{4\mathbf{i} + 12\mathbf{j}\}\ \text{m}$$

$$\mathbf{F} = \{0.4588\mathbf{i} + 1.376\mathbf{j} - 1.376\mathbf{k}\}\ \text{kN}$$

$$\mathbf{M}_O = \mathbf{r}_A \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 12 \\ 0.4588 & 1.376 & -1.376 \end{vmatrix}$$

$$= [0(-1.376) - 12(1.376)]\mathbf{i} - [0(-1.376) - 12(0.4588)]\mathbf{j} + [0(1.376) - 0(0.4588)]\mathbf{k}$$

$$= \{-16.5\mathbf{i} + 5.51\mathbf{j}\}\ \text{kN}\cdot\text{m}$$

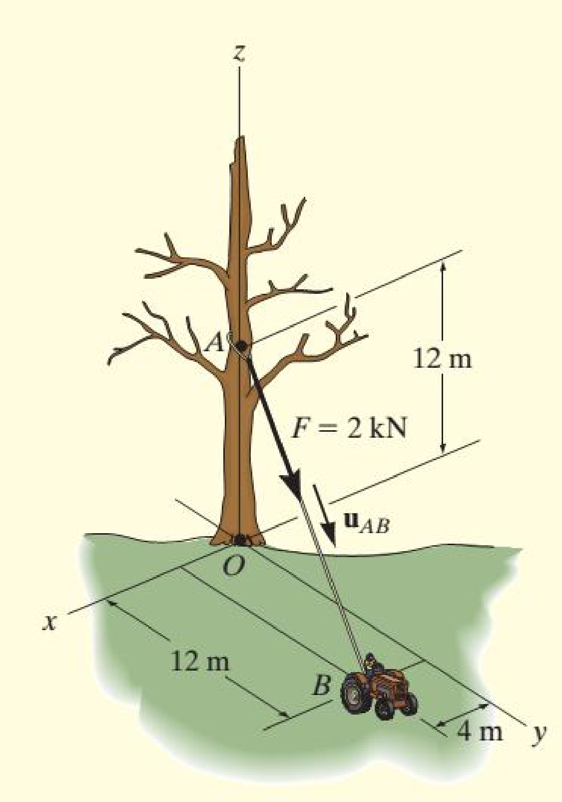
力对点之矩

例：求力F对O的矩。u_{AB}是单位向量。

解：

$$\mathbf{r}_A = \{12\mathbf{k}\}\ \text{m} \quad \text{and} \quad \mathbf{r}_B = \{4\mathbf{i} + 12\mathbf{j}\}\ \text{m}$$

$$\mathbf{F} = \{0.4588\mathbf{i} + 1.376\mathbf{j} - 1.376\mathbf{k}\}\ \text{kN}$$



力对点之矩

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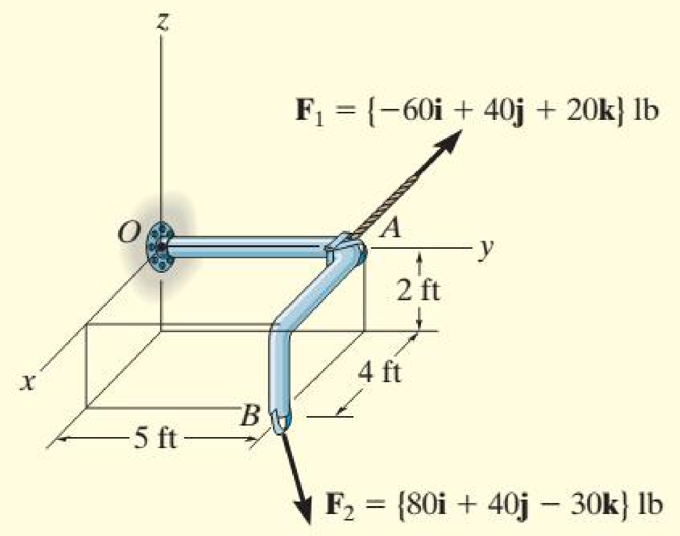
例：求力F对O的矩。



解：

$$\mathbf{r}_A = \{5\mathbf{j}\}\ \text{ft}$$

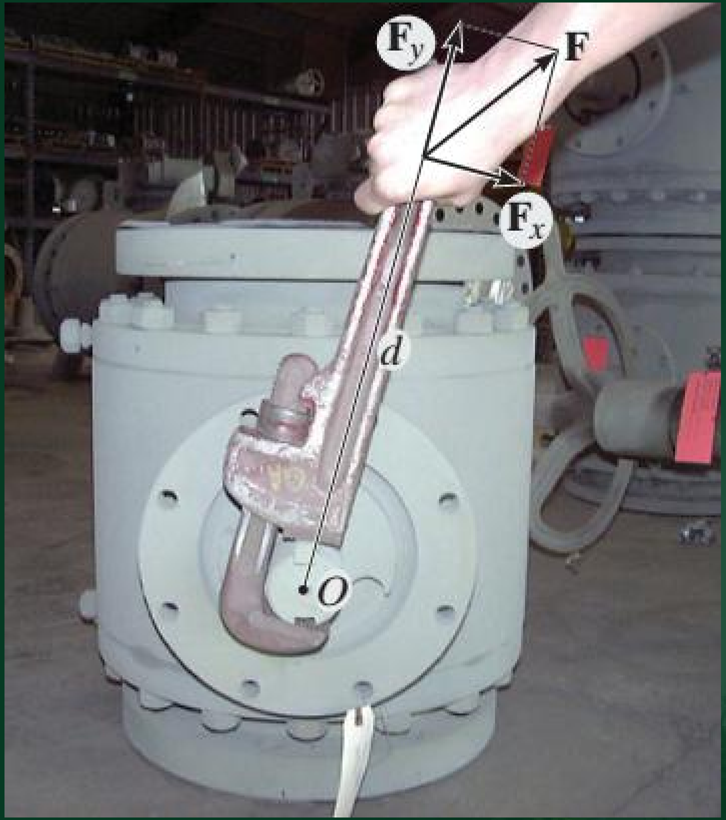
$$\mathbf{r}_B = \{4\mathbf{i} + 5\mathbf{j} - 2\mathbf{k}\}\ \text{ft}$$

$$\begin{aligned}
\mathbf{M}_O &= \Sigma(\mathbf{r} \times \mathbf{F}) \\
&= \mathbf{r}_A \times \mathbf{F}_1 + \mathbf{r}_B \times \mathbf{F}_2 \\
&= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 5 & 0 \\ -60 & 40 & 20 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 5 & -2 \\ 80 & 40 & -30 \end{vmatrix} \\
&= [5(20) - 0(40)]\mathbf{i} - [0]\mathbf{j} + [0(40) - (5)(-60)]\mathbf{k} \\
&\quad + [5(-30) - (-2)(40)]\mathbf{i} - [4(-30) - (-2)(80)]\mathbf{j} + [4(40) - 5(80)]\mathbf{k} \\
&= \{30\mathbf{i} - 40\mathbf{j} + 60\mathbf{k}\}\ \text{lb} \cdot \text{ft}
\end{aligned}$$

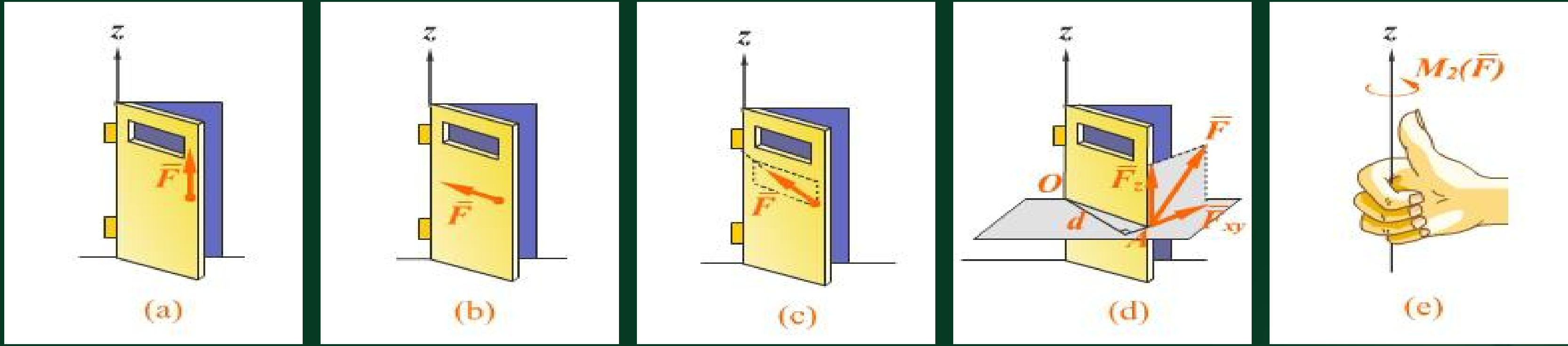
合力矩定理

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \mathbf{r} \times (\mathbf{F}_1 + \mathbf{F}_2) = \mathbf{r} \times \mathbf{F}_1 + \mathbf{r} \times \mathbf{F}_2$$

合理应用合力矩定理，
问题会变得简单一些。



力对轴之矩



$$M_z(\vec{F}) = M_o(\vec{F}_{xy}) = \pm F_{xy} \cdot h$$

力与轴相交或与轴平行（力与轴在同一平面内），力对该轴的矩为零。

力对轴之矩

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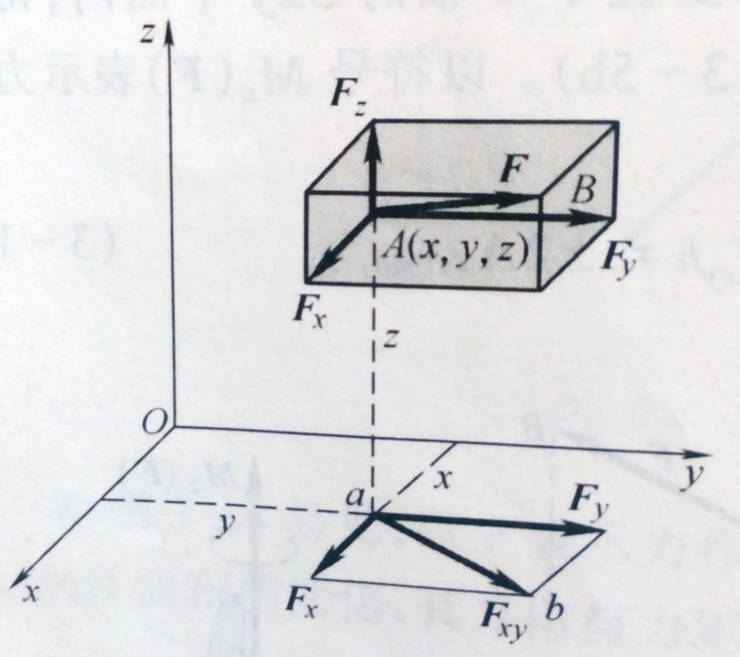
力对点的矩与力对过该点的轴的矩的关系

已知：力 $\vec{F}$ 在三根轴上的分力 $\vec{F}_x$，$\vec{F}_y$，$\vec{F}_z$， 力 $\vec{F}$ 作用点的坐标 x, y, z

求：力 $\vec{F}$ 对 x, y, z轴的矩

$$M_x(\vec{F}) = M_x(\vec{F}_x) + M_x(\vec{F}_y) + M_x(\vec{F}_z)$$
$$= 0 - F_y \cdot z + F_z \cdot y$$
$$= F_z \cdot y - F_y \cdot z$$

$$M_y(\vec{F}) = M_y(\vec{F}_x) + M_y(\vec{F}_y) + M_y(\vec{F}_z)$$



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求： 力F对 x, y, z轴的矩

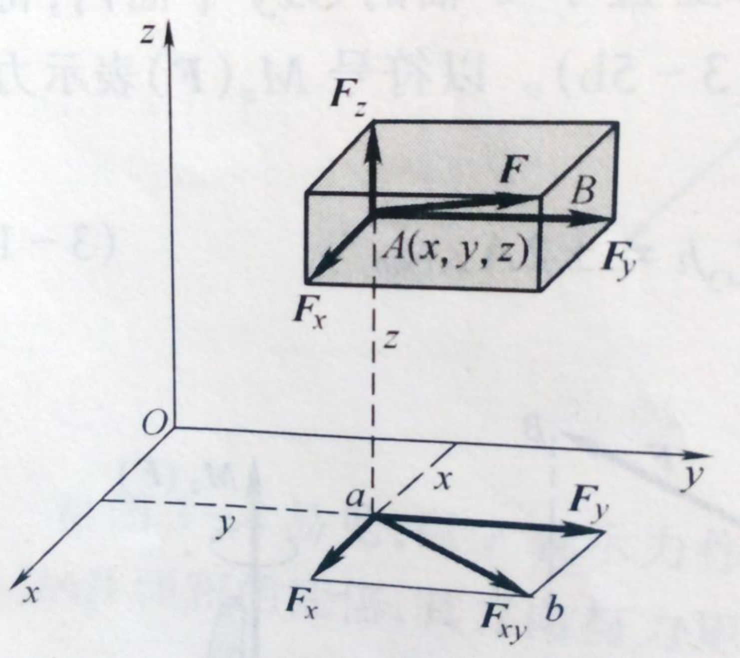
$$M_x(\vec{F}) = F_z \cdot y - F_y \cdot z$$

$$M_y(\vec{F}) = M_y(\vec{F}_x) + M_y(\vec{F}_y) + M_y(\vec{F}_z)$$

$$= F_x \cdot z + 0 - F_z \cdot x = F_x \cdot z - F_z \cdot x$$

$$M_z(\vec{F}) = M_z(\vec{F}_x) + M_z(\vec{F}_y) + M_z(\vec{F}_z)$$

$$= F_y \cdot x - F_x \cdot y$$



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求：力F对 x, y, z轴的矩

$$M_x(\vec{F}) = F_z \cdot y - F_y \cdot z$$

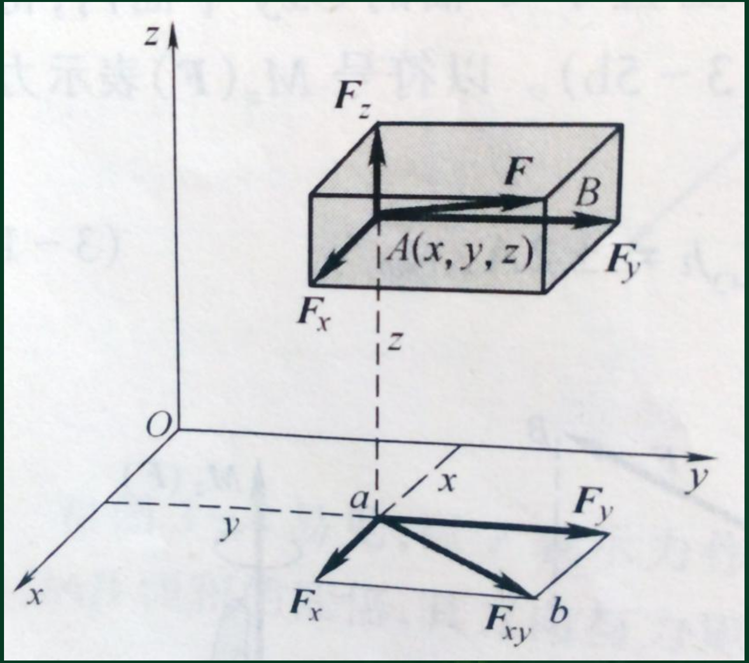
$$M_y(\vec{F}) = F_x \cdot z - F_z \cdot x$$

$$M_z(\vec{F}) = F_y \cdot x - F_x \cdot y$$

$$\left[\vec{M}_o(\vec{F})\right]_x = yF_z - zF_y = M_x(\vec{F})$$

$$\left[\vec{M}_o(\vec{F})\right]_y = zF_x - xF_z = M_y(\vec{F})$$

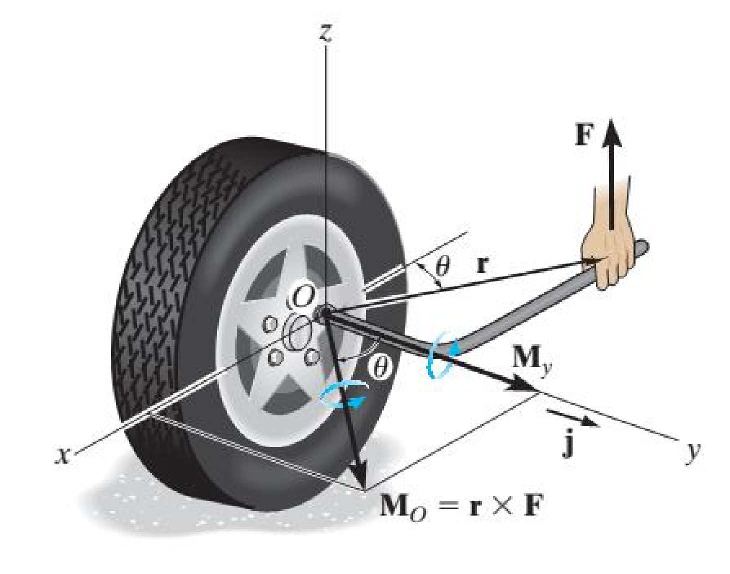
$$\left[\vec{M}_o(\vec{F})\right]_z = xF_y - yF_x = M_z(\vec{F})$$



力对轴之矩

$$M_a(\vec{F}) = \vec{u}_a \cdot \left[\vec{M}_o(\vec{F}) \right]$$

$$M_a = \vec{u}_a \cdot \left(\vec{r} \times \vec{F} \right)$$



力对轴之矩

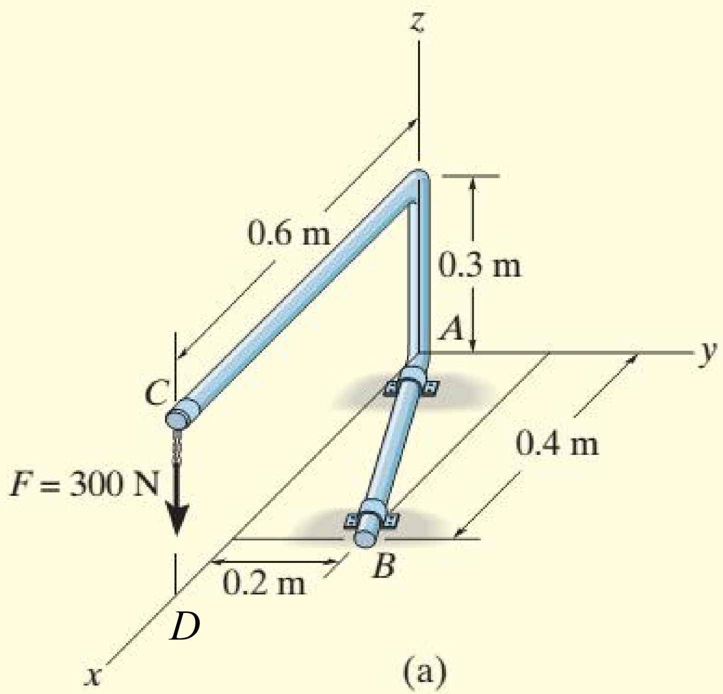
求力F对AB轴的矩

$$\mathbf{r}_D = \{0.6\mathbf{i}\}\ \text{m}$$

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{\{0.4\mathbf{i} + 0.2\mathbf{j}\}\ \text{m}}{\sqrt{(0.4\ \text{m})^2 + (0.2\ \text{m})^2}} = 0.8944\mathbf{i} + 0.4472\mathbf{j}$$

$$\mathbf{F} = \{-300\mathbf{k}\}\ \text{N}$$

$$M_{AB} = \vec{u}_B \cdot \left(\vec{r}_D \times \vec{F}\right)$$



(a)

力对轴之矩

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求力F对AB轴的矩

$$\mathbf{r}_D = \{0.6\mathbf{i}\}\ \text{m}$$

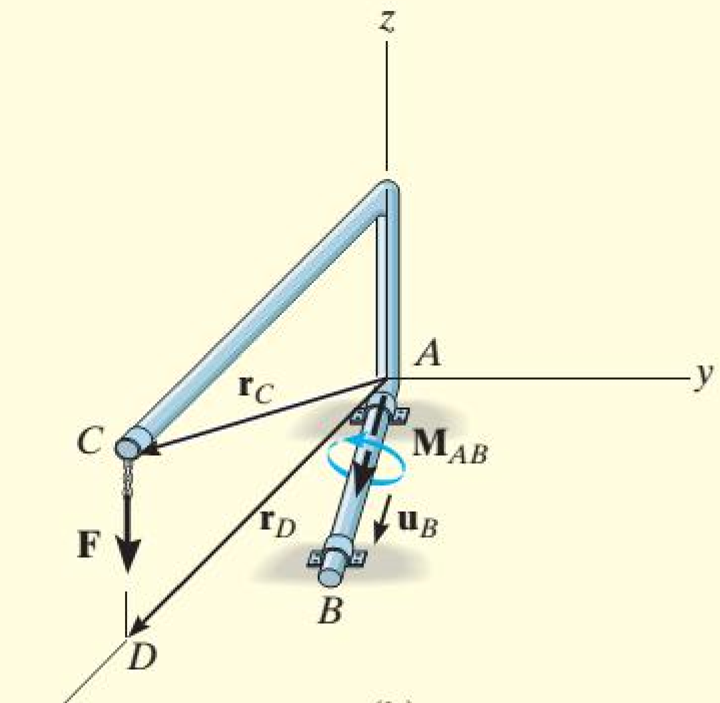
$$\mathbf{F} = \{-300\mathbf{k}\}\ \text{N}$$

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{\{0.4\mathbf{i} + 0.2\mathbf{j}\}\ \text{m}}{\sqrt{(0.4\ \text{m})^2 + (0.2\ \text{m})^2}} = 0.8944\mathbf{i} + 0.4472\mathbf{j}$$

$$M_{AB} = \mathbf{u}_B \cdot (\mathbf{r}_D \times \mathbf{F}) = \begin{vmatrix} 0.8944 & 0.4472 & 0 \\ 0.6 & 0 & 0 \\ 0 & 0 & -300 \end{vmatrix}$$

$$= 0.8944[0(-300) - 0(0)] - 0.4472[0.6(-300) - 0(0)] + 0[0.6(0) - 0(0)]$$

$$= 80.50\ \text{N}\cdot\text{m}$$



(b)

力对轴之矩

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已知：***F*, *l*, *a*, *θ*.** 手柄在*Axy*平面内，力F垂直于*y*轴

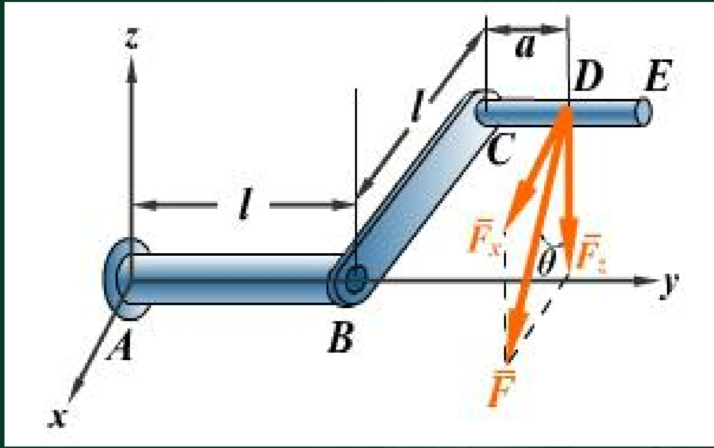
求：$M_x\left(\vec{F}\right), M_y\left(\vec{F}\right), M_z\left(\vec{F}\right)$

解：把力*F*分解如图

$$M_x\left(\vec{F}\right) = -F\left(l+a\right)\cos\theta$$

$$M_y\left(\vec{F}\right) = -Fl\cos\theta$$

$$M_z\left(\vec{F}\right) = -F\left(l+\alpha\right)\sin\theta$$



谢谢大家！