石家莊鐵道大學
SHIJIAZHUANG TIEDAO UNIVERSITY

在线开放课程

理论力学

平面力对点之矩与平面力偶

平面力偶

主讲 ： 郭树起

平面力偶

石家庄铁道大学 SHIJIAZHUANG TIEDAO UNIVERSITY

在线开放课程

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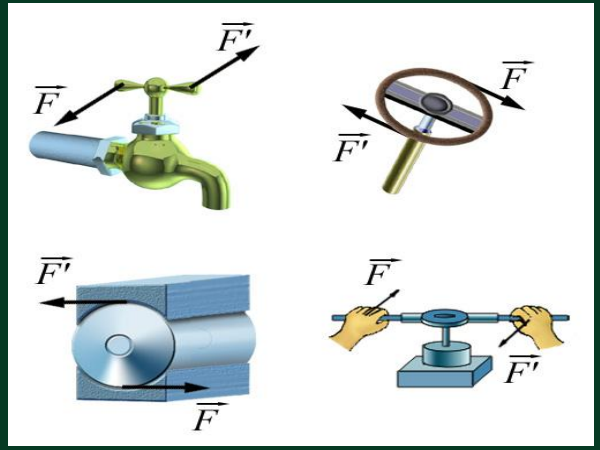
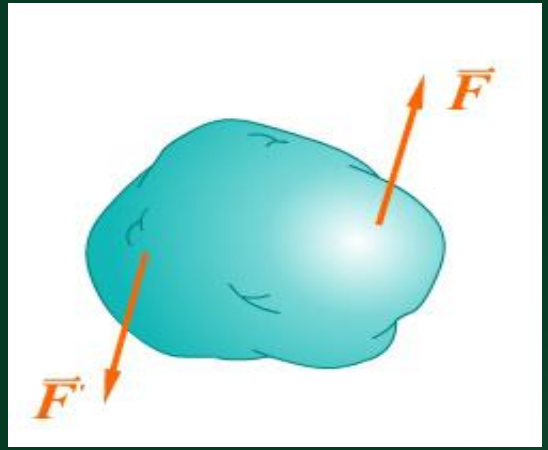
什么是力偶





由两个等值、反向、不共线的（平行）力组成的力系称为力偶，记作 $(\vec{F}, \vec{F}')$

什么是力偶

力偶中两力所在平面称为力偶作用面。

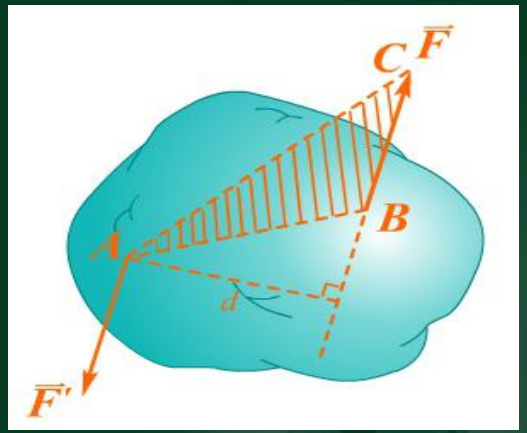
力偶两力之间的垂直距离称为力偶臂。

两个要素

a. 大小：力与力偶臂乘积

b. 方向：转动方向

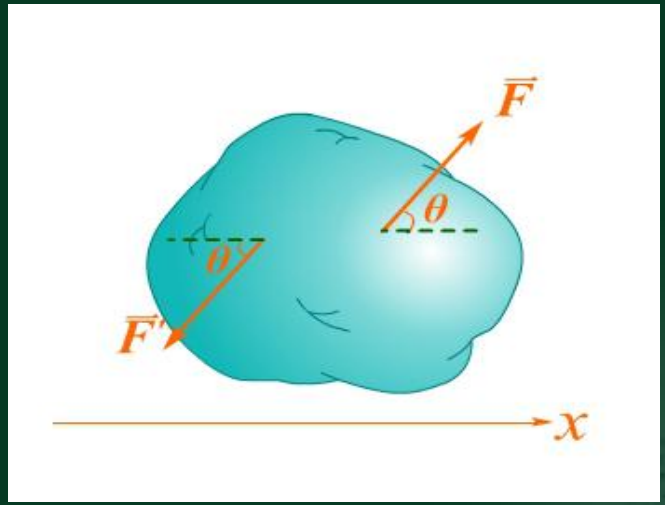
力偶矩



$$M = \pm F \cdot d = \pm 2 \cdot \frac{1}{2} \cdot F \cdot d = \pm 2\Delta ABC$$

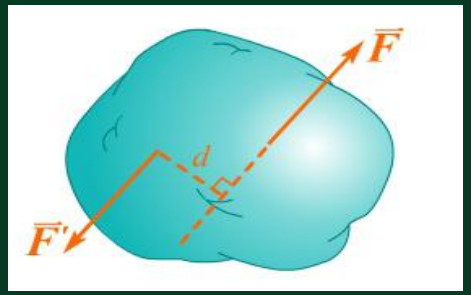
力偶与力偶矩的性质

1. 力偶在任意坐标轴上的投影等于零。



2. 力偶对任意点取矩都等于力偶矩，不因矩心的改变而变。

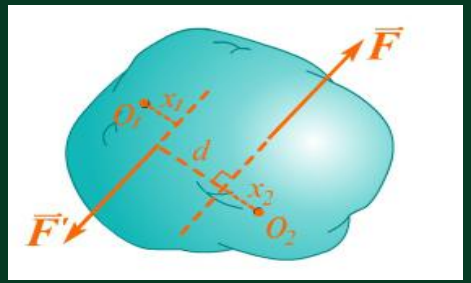
力偶与力偶矩的性质



$$M = F \cdot d$$

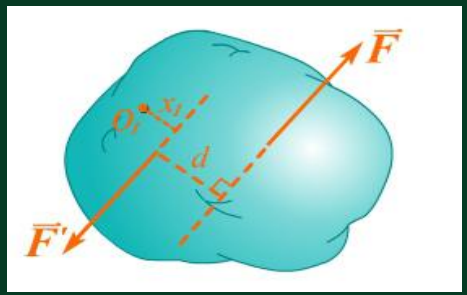
$$M_{O_1}\left(\vec{F},\vec{F}'\right) = M_{O_1}\left(\vec{F}\right) + M_{O_1}\left(\vec{F}'\right)$$

$$= F \cdot \left(d + x_1\right) - F \cdot x_1 = Fd$$



$$M_{O_2}\left(\vec{F},\vec{F}'\right) = F' \cdot \left(d + x_2\right) - F \cdot x_2$$

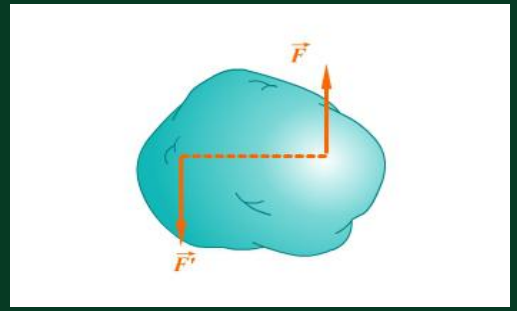
$$= F'd = Fd$$



力矩的符号 $M_O\left(\vec{F}\right)$ 力偶矩的符号 **M**

力偶与力偶矩的性质

3. 只要保持力偶矩不变，力偶可在其作用面内任意移转，且可以同时改变力偶中力的大小与力臂的长短，对刚体的作用效果不变。



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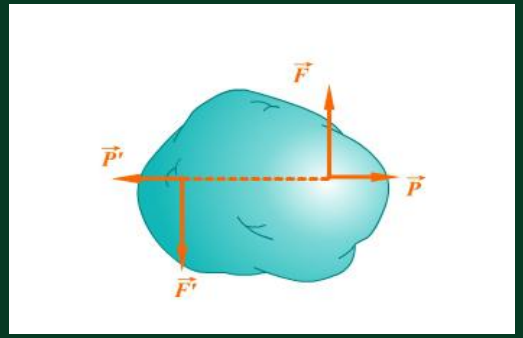

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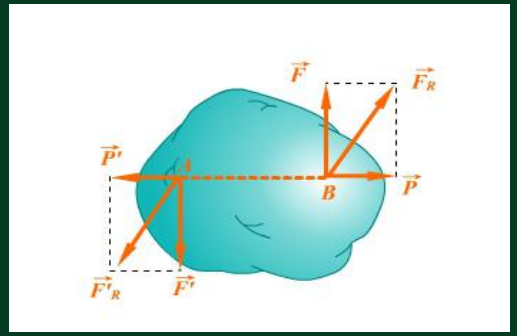

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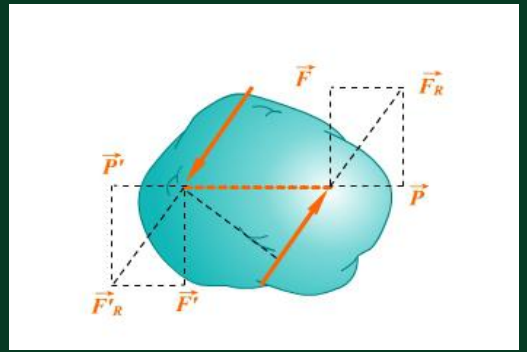


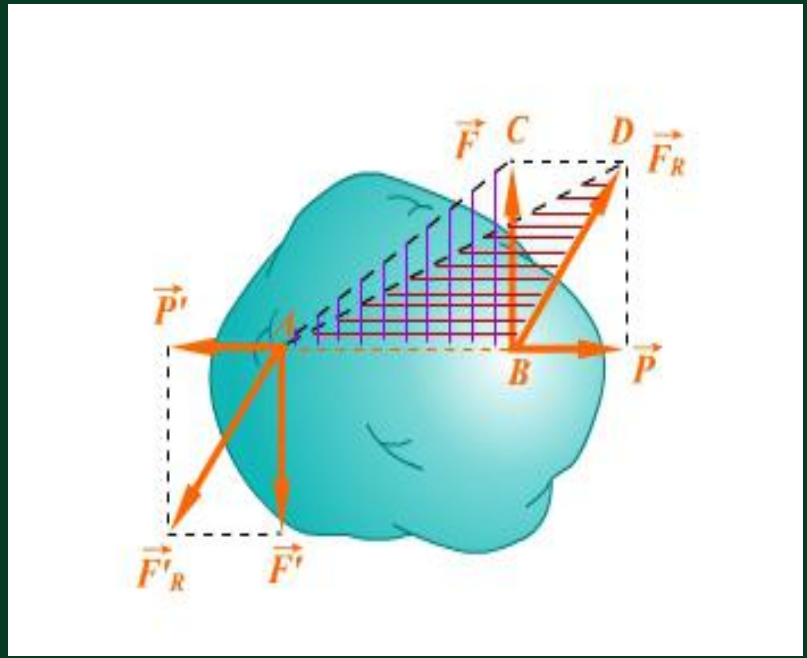
力偶与力偶矩的性质

$$M\left(\vec{F},\vec{F}'\right)=Fd=2\Delta ABC$$

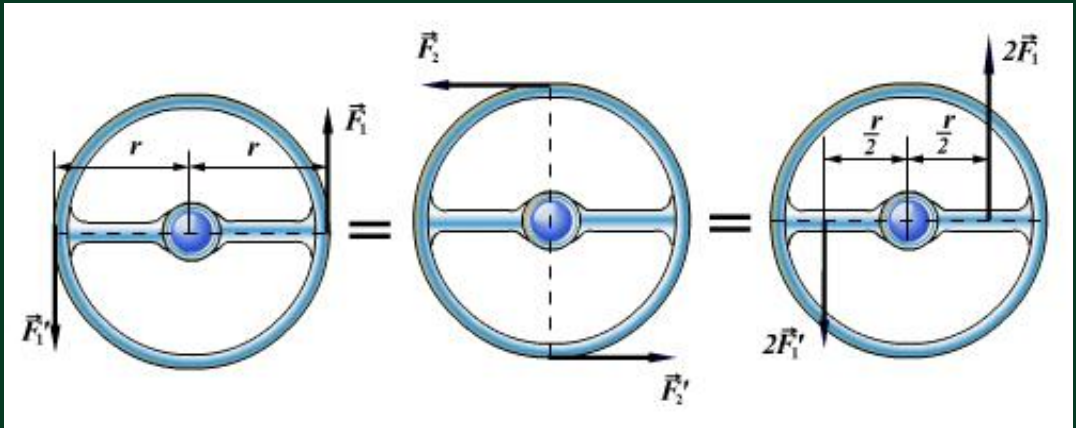
$$M\left(\vec{F}_R,\vec{F}'_R\right)=F_Rd_1=2\Delta ABD$$

$$\Delta ABC\,?\,\Delta ABD$$

$$\Delta ABC=\Delta ABD$$

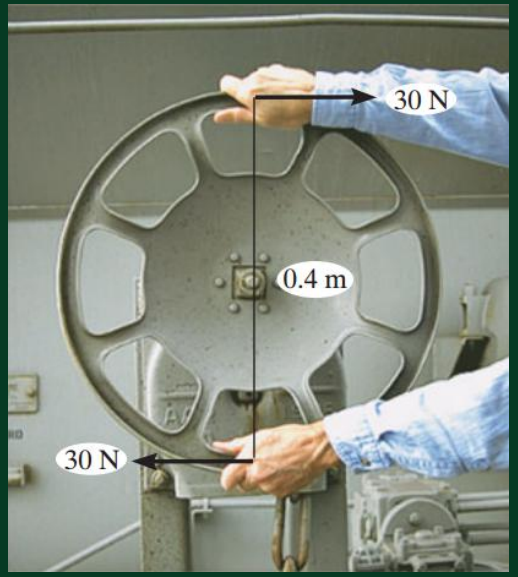


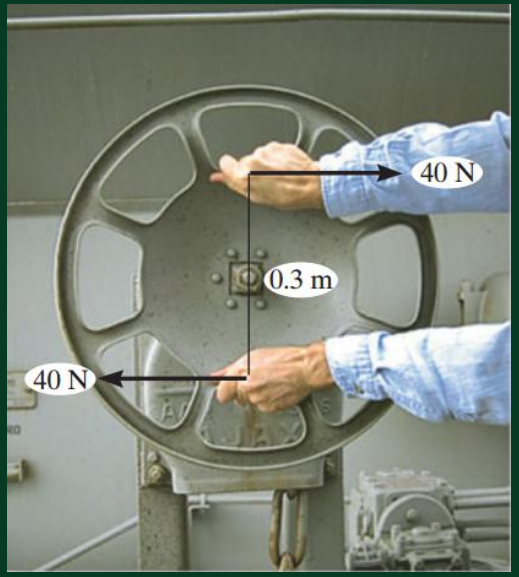
力偶与力偶矩的性质



4. 力偶没有合力，力偶只能由力偶来平衡。

力偶与力偶矩的性质





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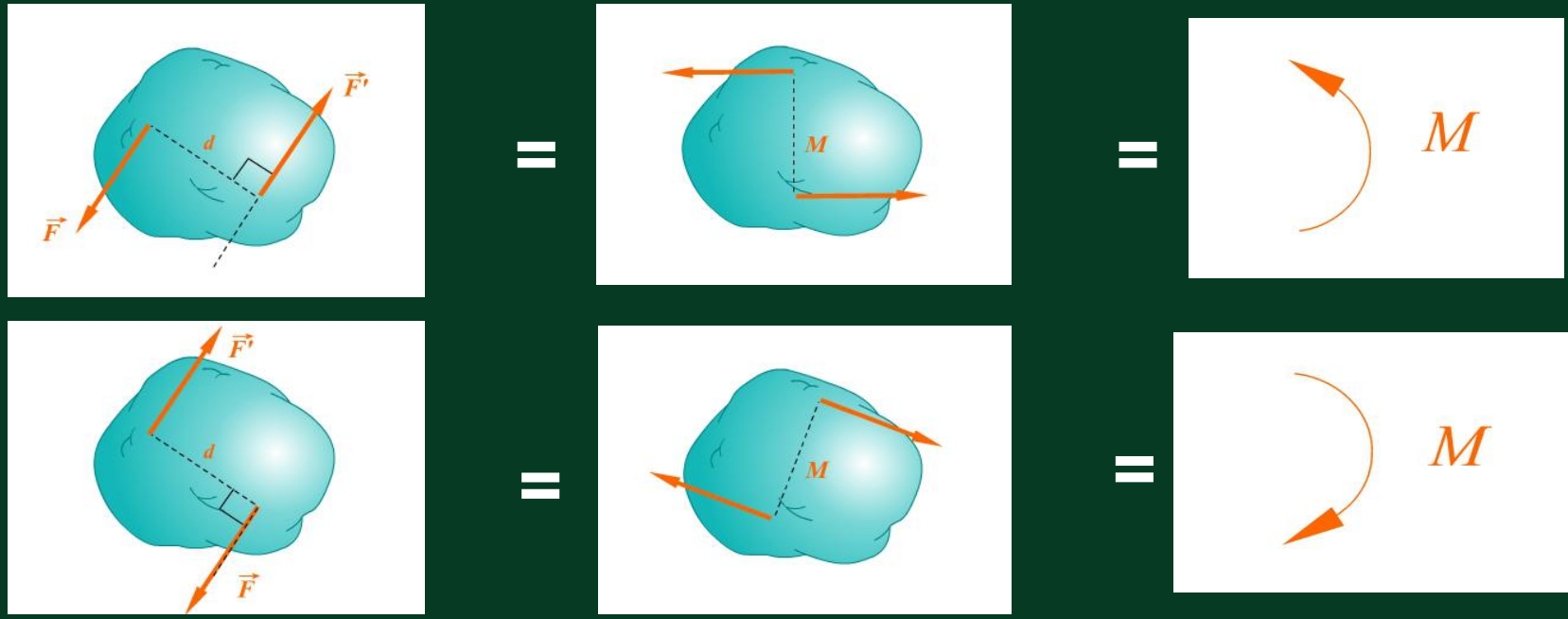
力偶与力偶矩的性质

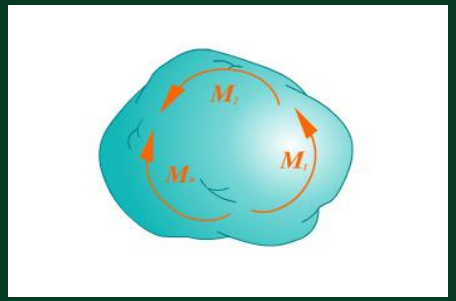


平面力偶系的合成与平衡条件

已知：$M_1, M_2, \cdots M_n$;

任选一段距离d $\quad \dfrac{M_1}{d} = F_1 \quad M_1 = F_1 d \quad \dfrac{M_2}{d} = F_2 \quad M_2 = F_2 d$

$$\left|\frac{M_n}{d}\right| = F_n \qquad M_n = -F_n d$$



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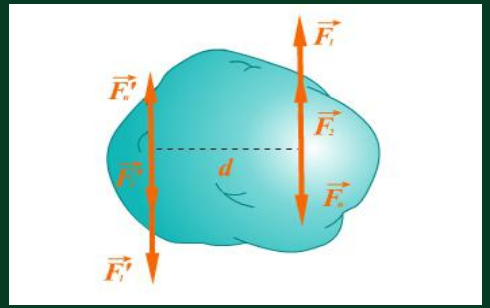

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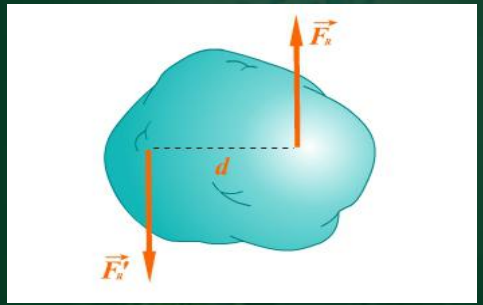


平面力偶系的合成与平衡条件

$$F_R = F_1 + F_2 + \cdots - F_n$$

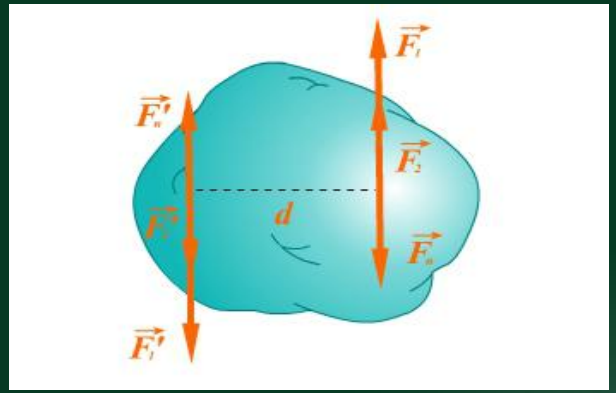
$$F'_R = F'_1 + F'_2 + \cdots - F'_n$$

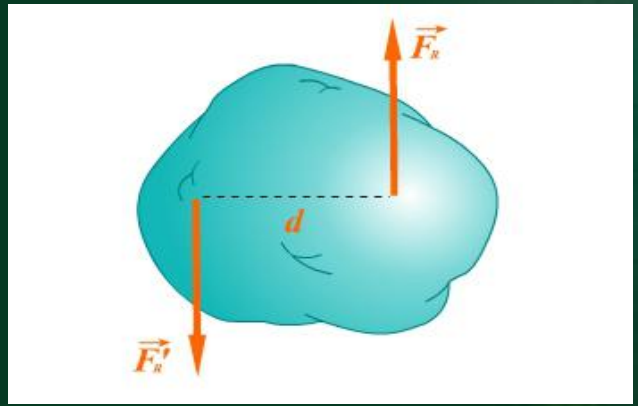
$$M = F_R d$$

$$= F_1 d + F_2 d + \cdots - F_n d$$

$$= M_1 + M_2 + \cdots M_n$$

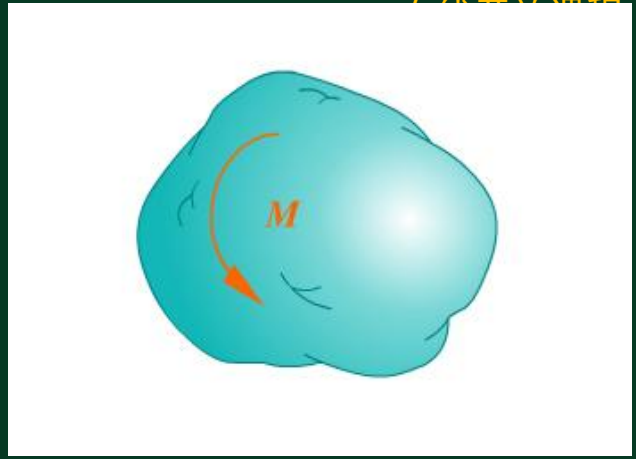
$$M = \sum_{i=1}^{n} M_i = \Sigma M_i$$





平面力偶系的合成与平衡条件

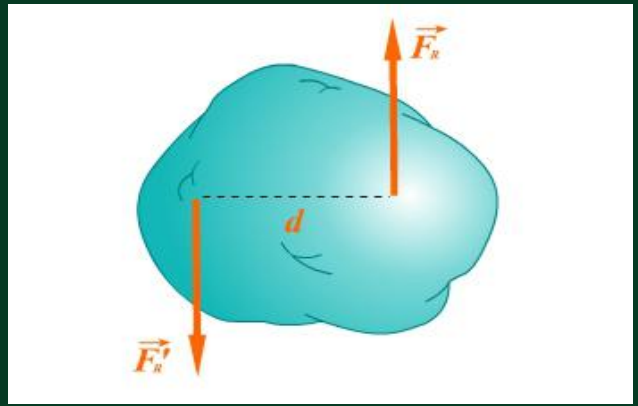
$$M=\sum_{i=1}^{n}M_i=\sum M_i$$



平面力偶系平衡的充要条件

$$M=0$$

即 $\sum M_i=0$

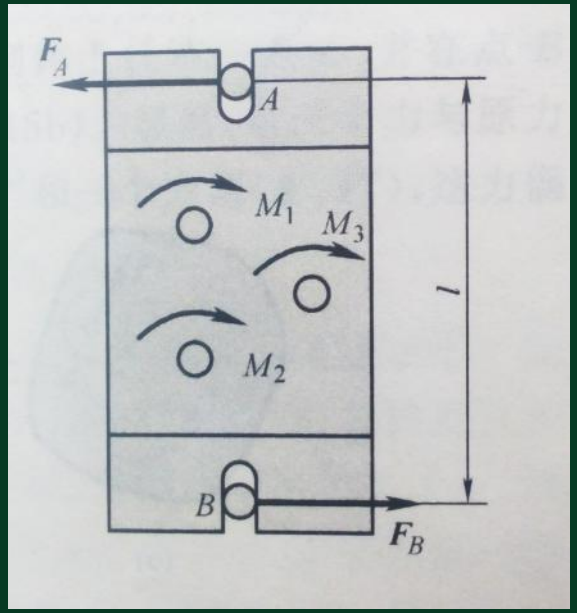


平面力偶系的合成与平衡条件

已知 $M_1 = M_2 = 10\,\mathrm{N}\cdot\mathrm{m}$, $M_3 = 20\,\mathrm{N}\cdot\mathrm{m}$, $l = 200\,\mathrm{mm}$;

求： 光滑螺柱*AB*所受水平力。

解：由力偶只能由力偶平衡的性质，其受力图为



$$\sum M = 0$$

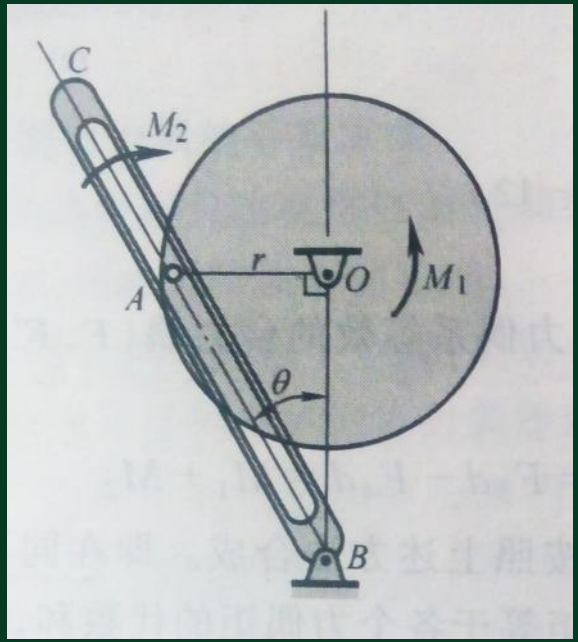
$$F_A l - M_1 - M_2 - M_3 = 0$$

$$F_A = F_B = \frac{M_1 + M_2 + M_3}{l} = 200\,\mathrm{N}$$

平面力偶系的合成与平衡条件

已知 $M_1 = 2\mathbf{kN}\cdot\mathbf{m}, OA = r = 0.5\mathbf{m}, \theta = 30^\circ;$

求：平衡时的 M_2 及铰链*O*，*B*处的约束力。



平面力偶系的合成与平衡条件

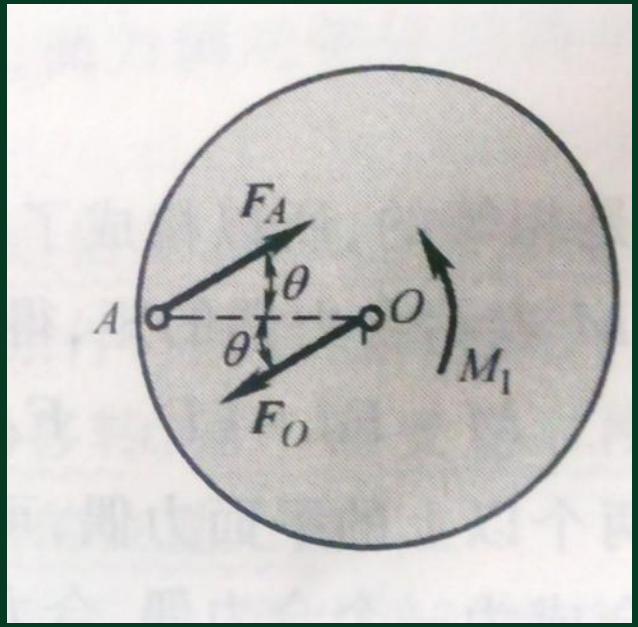
已知 $M_1 = 2\mathbf{kN}\cdot\mathbf{m}, OA = r = 0.5\mathbf{m}, \theta = 30^\circ;$

求：平衡时的 M_2 及铰链*O*，*B*处的约束力。

取圆盘为研究对象，画受力图

$$\sum M = 0 \quad M_1 - F_A \cdot r\sin\theta = 0$$

解得 $F_O = F_A = 8\mathbf{kN}$



平面力偶系的合成与平衡条件

已知 $M_1 = 2\mathrm{kN}\cdot\mathrm{m}, OA = r = 0.5\mathrm{m}, \theta = 30^\circ;$

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$$\sum M = 0 \quad M_1 - F_A \cdot r\sin\theta = 0$$

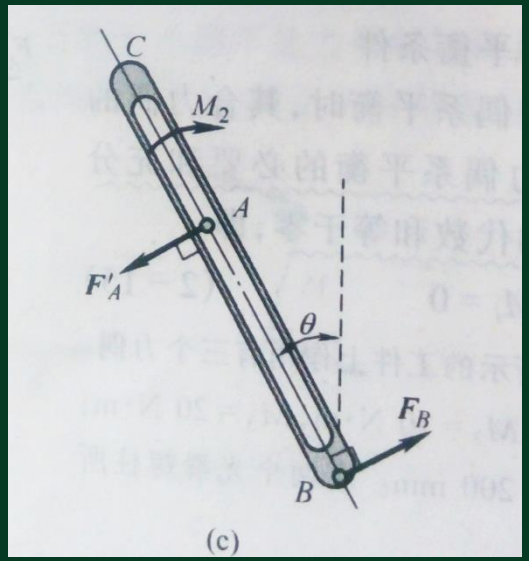
解得 $F_O = F_A = 8\mathrm{kN}$

取杆*BC*，画受力图。

$$\sum M = 0 \quad F_A' \cdot \frac{r}{\sin\theta} - M_2 = 0$$

解得 $M_2 = 8\mathrm{kN}\cdot\mathrm{m}$

$F_B = F_A = 8\mathrm{kN}$



謝謝大家！