石家莊鐵道大學
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网络精品课程

高等数学下

多元函数微分学及其应用

多元复合函数求导法(1)

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目录

- 链式法则(1)；
- 链式法则(2)；
- 全微分的形式不变性。

链式法则(1)

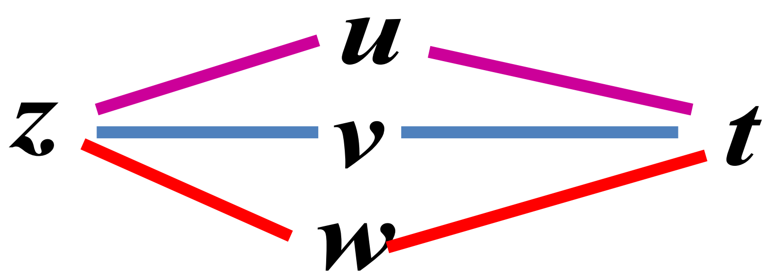
定理　如果函数 $u=\phi(t)$ 及 $v=\psi(t)$ 都在点 t 可导，函数 $z=f(u,v)$ 在对应点 (u,v) 具有连续偏导数，则复合函数 $z=f[\phi(t),\psi(t)]$ 在对应点 t 可导，且其导数可用下列公式计算：

$$\frac{dz}{dt}=\frac{\partial z}{\partial u}\frac{du}{dt}+\frac{\partial z}{\partial v}\frac{dv}{dt}.$$

链式法则(1)

上定理的结论可推广到中间变量多于两个的情况.

$$\frac{dz}{dt}=\frac{\partial z}{\partial u}\frac{du}{dt}+\frac{\partial z}{\partial v}\frac{dv}{dt}+\frac{\partial z}{\partial w}\frac{dw}{dt}$$



以上公式中的导数 $\frac{dz}{dt}$ 称为**全导数.**

链式法则(1)

定理　如果$u=u(x, y)$及$v=v(x, y)$在点(x, y)对x和y的偏导数都存在, 且函数$z=f(u, v)$在对应点(u, v)可微, 则复合函数$z=f(u(x, y), v(x, y))$在点(x, y)的两个偏导数都存在, 且

$$\frac{\partial z}{\partial x}=\frac{\partial f}{\partial u}\frac{\partial u}{\partial x}+\frac{\partial f}{\partial v}\frac{\partial v}{\partial x},$$

$$\frac{\partial z}{\partial y}=\frac{\partial f}{\partial u}\frac{\partial u}{\partial y}+\frac{\partial f}{\partial v}\frac{\partial v}{\partial y}.$$

链式法则(1)

几种特殊情形: 定理讲的是2个中间变量, 2个自变量的情形, 但其思想方法完全适用于其它情形:

设$u=f(x, y, z)$, $x=x(s, t)$, $y=y(s, t)$, $z=z(s, t)$, 则

$$\frac{\partial u}{\partial s}=\frac{\partial u}{\partial x}\cdot\frac{\partial x}{\partial s}+\frac{\partial u}{\partial y}\cdot\frac{\partial y}{\partial s}+\frac{\partial u}{\partial z}\cdot\frac{\partial z}{\partial s};$$

$$\frac{\partial u}{\partial t}=\frac{\partial u}{\partial x}\cdot\frac{\partial x}{\partial t}+\frac{\partial u}{\partial y}\cdot\frac{\partial y}{\partial t}+\frac{\partial u}{\partial z}\cdot\frac{\partial z}{\partial t}.$$

链式法则(2)

特殊地 $z=f(u,x,y)$ 其中 $u=\phi(x,y)$

即 $z=f[\phi(x,y),x,y]$,

$$\frac{\partial z}{\partial x}=\frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial f}{\partial x},\qquad \frac{\partial z}{\partial y}=\frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial f}{\partial y}.$$

两者的区别

把复合函数 $z=f[\phi(x,y),x,y]$ 中的 y 看作不变而对 x 的偏导数

把 $z=f(u,x,y)$ 中的 u 及 y 看作不变而对 x 的偏导数

全微分的形式不变性

设函数$z=f(u, v)$具有连续偏导数, 则有全微分

$$\mathrm{d}z=\frac{\partial f}{\partial u}\mathrm{d}u+\frac{\partial f}{\partial v}\mathrm{d}v.$$

当$u=u(x, y)$, $v=v(x, y)$时, 有

$$\mathrm{d}z=\frac{\partial z}{\partial x}\mathrm{d}x+\frac{\partial z}{\partial y}\mathrm{d}y=(\frac{\partial f}{\partial u}\frac{\partial u}{\partial x}+\frac{\partial f}{\partial v}\frac{\partial v}{\partial x})\mathrm{d}x+(\frac{\partial f}{\partial u}\frac{\partial u}{\partial y}+\frac{\partial f}{\partial v}\frac{\partial v}{\partial y})\mathrm{d}y$$

$$=\frac{\partial f}{\partial u}(\frac{\partial u}{\partial x}\mathrm{d}x+\frac{\partial u}{\partial y}\mathrm{d}y)+\frac{\partial f}{\partial v}(\frac{\partial v}{\partial x}\mathrm{d}x+\frac{\partial v}{\partial y}\mathrm{d}y)$$

$$=\frac{\partial f}{\partial u}\mathrm{d}u+\frac{\partial f}{\partial v}\mathrm{d}v.$$

即不论(u, v)为自变量还是中间变量, 均有$\mathrm{d}z=\frac{\partial f}{\partial u}\mathrm{d}u+\frac{\partial f}{\partial v}\mathrm{d}v.$

此性质称为一阶全微分形式不变性.